Comments on the last development in constructing the amplitude for the radiative muon capture

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Abstract

It has recently been claimed by Cheon and Cheoun that the discrepancy between the experimental value of the induced pseudoscalar g_P, obtained recently at TRIUMF from a measurement of the radiative muon capture by proton, and its value predicted by using PCAC and pion pole dominance can be explained by a contact term generated from a simple pion-nucleon Lagrangian. We show in our comment that this claim is ill founded.

In a recent preprint [1], Cheon and Cheoun claim to remove the discrepancy between the value of the induced pseudoscalar g_P obtained in the TRIUMF experiment [2] investigating the radiative muon capture (RMC) by proton and its prediction from the PCAC and the pion pole dominance. They start from a linear σ-model Lagrangian of the $\pi - N$ system and derive a general equation for the axial current and its divergence. Then they introduce into the Lagrangian the electromagnetic coupling by the minimal substitution $\partial_\mu \longrightarrow D_\mu = \partial_\mu - ie\epsilon_\mu$ and using a standard local transformation of the nucleon $\Psi(x)$ and pion $\phi_a(x)$ fields,

$$\Psi \rightarrow \Psi' = (1 + i\gamma_5 \frac{\vec{\tau}}{2} \cdot \vec{\eta}(x))\Psi\,, \qquad \vec{\phi} \rightarrow \vec{\phi}' = \vec{\phi} - f_\pi \eta(\vec{x})\,, \tag{1}$$

and the above mentioned equation of motion, they derive the axial current $A_a^\mu(x)$ in the presence of the electromagnetic field. In the final Eq. (22) for the axial current, besides the well known terms a large contact term (the last term in the r. h. s. of the equation) appears which, according to Cheon and Cheoun, should resolve the problem.

The basic equation of Ref. [1] is Eq. (14)

$$\widetilde{\mathcal{L}}_0 = \bar{\Psi}[i\gamma^\mu D_\mu + g f_\pi exp(\frac{i}{f_\pi}\gamma_5 \vec{\tau} \cdot \vec{\phi})]\Psi - \bar{\Psi}\gamma^\mu \gamma_5 \frac{\vec{\tau}}{2}\Psi \cdot (D_\mu \vec{\eta}) + \frac{1}{2}(D_\mu \vec{\phi})^2 - (D^\mu \vec{\phi}) \cdot f_\pi (D_\mu \vec{\eta})\,. \tag{2}$$

Actually, it is the 3rd term in the r. h. s. of this equation $\sim$ $(D_\mu \vec{\eta})$ which leads to the large contact contribution. But it can be seen that under the local transformations (1), the correct term is

$$-\Psi\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2}\Psi \cdot (\partial_\mu \vec{\eta}) = -\Psi\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2}\Psi \cdot (D_\mu \vec{\eta}) - \Psi\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2}\Psi \cdot (ie\epsilon_\mu \vec{\eta})\,. \tag{3}$$

Using Eqs. (13) [1] defining the current and its divergence and Eq. (14) [1] with the new term $\sim$ $(ie\epsilon_\mu \vec{\eta})$ from our Eq. (3) included we can see that instead of the current $A_a^\mu(x)$ with the divergence equal zero we now have [cf. Eqs. (16) [1]]

$$D_\mu^{(+)}\, A_a^\mu = \Psi\gamma^\mu\gamma_5 \frac{\tau_a}{2}\Psi(ie\epsilon_\mu)\,. \tag{4}$$

Taking into account Eq. (4) and following the derivation of the axial current in [1] we get the same Eq. (22) but with the last term in the r. h. s. multiplied by the factor $(g_A - 1)/g_A \approx 0.20$

$$\frac{g_A - 1}{g_A} \frac{e g_P(q^2)}{2 m m_\mu}\, q^\mu\, [\bar{\Psi}(x)\, \epsilon_\alpha \gamma^\alpha \gamma_5 \frac{\tau_a}{2}\, \Psi(x)]\,. \tag{5}$$

So the effect of the questioned contact term is strongly suppressed.

Actually, the appearance of such a term in the leading order in the RMC amplitude constructed at the tree level from a chiral invariant Lagrangian would contradict the low energy theorem prediction for the hadron part of the RMC amplitude and in essence the current algebras and PCAC. It appears in [1] due to an artificial introduction of a piece of the pseudovector $\pi - N$ coupling via Eq. (18) which provides the factor $(g_A - 1)/g_A$.

Otherwise, the current (5) should be absent in any model based on the chiral Lagrangian with any mixing of the $\pi - N$ couplings (see below).

We have recently derived [3] the RMC amplitude from a chiral invariant Lagrangian of the $N\pi\rho\, a_1\, \omega$ system constructed within the framework of the hidden local symmetry approach [4–7]. The contact term of the type just discussed above appears in our Eq. (3.11) but it is cancelled in the leading order by another contact term of Eq. (3.32) and only higher order terms in k and q survive. The chiral Lagrangian of the $\pi - N$ system can be obtained from our Lagrangian Eq. (2.1) by performing the limit $m_B = \infty$ for all heavy mesons. As an example, we give here the $N\pi$ Lagrangian with the pseudovector $\pi - N$ coupling

$$\begin{aligned}
\mathcal{L}_{N\pi} \,=\, & -\bar{\Psi}\,\gamma_\mu\partial_\mu\,\Psi \,-\, M\,\bar{\Psi}\,\Psi \,-\, i\frac{1}{4f_\pi^2}\,\bar{\Psi}\,\gamma_\mu\vec{\tau}\,\Psi\cdot(\vec{\Pi}\times\partial_\mu\vec{\Pi}) \\
& -i\frac{e}{2}\,\bar{\Psi}\,\gamma_\mu\vec{\mathcal{V}}_\mu\cdot\vec{\tau}\,\Psi \,-\, i\frac{e}{2f_\pi}\,\bar{\Psi}\,\gamma_\mu\vec{\tau}\,\Psi\cdot(\vec{\Pi}\times\vec{\mathcal{A}}_\mu) \\
& -\frac{e}{4}\frac{\kappa_V}{2M}\,\bar{\Psi}\,\sigma_{\mu\nu}\vec{\tau}\,\Psi\cdot(\partial_\mu\vec{\mathcal{V}}_\nu \,-\, \partial_\nu\vec{\mathcal{V}}_\mu) \,+\, \frac{e^2}{4}\frac{\kappa_V}{2M}\,\bar{\Psi}\,\sigma_{\mu\nu}\vec{\tau}\,\Psi\cdot(\vec{\mathcal{V}}_\mu\times\vec{\mathcal{V}}_\nu) \\
& -i\frac{g_A}{2f_\pi}\,\bar{\Psi}\,\gamma_\mu\gamma_5\vec{\tau}\,\Psi\cdot(\partial_\mu\vec{\Pi}) \,-\, i\frac{eg_A}{2}\,\bar{\Psi}\,\gamma_\mu\gamma_5\vec{\tau}\,\Psi\cdot\vec{\mathcal{A}}_\mu \\
& -i\frac{eg_A}{2f_\pi}\,\bar{\Psi}\,\gamma_\mu\gamma_5\vec{\tau}\,\Psi\cdot(\vec{\Pi}\times\vec{\mathcal{V}}_\mu) \,+\, \mathcal{O}(|\Psi|^4\,,|\Pi|^3)\,,
\end{aligned} \tag{6}$$

$$\begin{aligned}
\mathcal{L}_\pi \,=\, & -\frac{1}{2}(\partial_\mu\vec{\Pi})^2 \,-\, ef_\pi(\vec{\mathcal{A}}_\mu\cdot\partial_\mu\vec{\Pi}) \,+\, e\,\vec{\mathcal{V}}_\mu\cdot(\vec{\Pi}\times\partial_\mu\vec{\Pi}) \\
& -e^2\,f_\pi\vec{\Pi}\cdot(\vec{\mathcal{V}}_\mu\times\vec{\mathcal{A}}_\mu) \,+\, \mathcal{O}(|\Pi|^4)\,.
\end{aligned} \tag{7}$$

The associated currents derived by the Glashow–Gell-Mann method [8] read

$$\begin{aligned}
\vec{J}_{V,\mu} \,=\, & -(\vec{\Pi}\times\partial_\mu\vec{\Pi}) \,-\, ef_\pi\,(\vec{\Pi}\times\vec{\mathcal{A}}_\mu) \,+\, \frac{i}{2}\,\bar{\Psi}\,\gamma_\mu\vec{\tau}\,\Psi \,-\, i\frac{g_A}{2f_\pi}\,\bar{\Psi}\,\gamma_\mu\gamma_5(\vec{\Pi}\times\vec{\tau})\,\Psi \\
& -\,\frac{\kappa_V}{4M}\,\partial_\nu[\bar{\Psi}\,\sigma_{\mu\nu}\vec{\tau}\,\Psi] \,-\, \frac{e}{2}\frac{\kappa_V}{2M}\,\bar{\Psi}\,\sigma_{\mu\nu}(\vec{\mathcal{V}}_\nu\times\vec{\tau})\,\Psi \,+\, \mathcal{O}(|\Psi|^4\,,|\Pi|^3)\,,
\end{aligned} \tag{8}$$

$$\begin{aligned}
\vec{J}_{A,\mu} \,=\, & f_\pi\,\partial_\mu\vec{\Pi} \,+\, ef_\pi\,(\vec{\Pi}\times\vec{\mathcal{V}}_\mu) \,+\, i\frac{g_A}{2}\,\bar{\Psi}\,\gamma_\mu\gamma_5\vec{\tau}\,\Psi \\
& -\,i\frac{1}{2f_\pi}\,\bar{\Psi}\,\gamma_\mu(\vec{\Pi}\times\vec{\tau})\,\Psi \,+\, \mathcal{O}(|\Psi|^4\,,|\Pi|^3)\,.
\end{aligned} \tag{9}$$

Here $\vec{\mathcal{V}}_\nu$ and $\vec{\mathcal{A}}_\mu$ are the external vector and axial fields.

The only seagull of the type (5) can be constructed using the last term of the Lagrangian (6) and the first term of the current (9), but it does not contribute to the RMC amplitude. The same situation can be seen after applying the Foldy-Dyson transformation to the Lagrangian (6)

$$\Psi \,=\, exp(-i\lambda\frac{g_A}{2f_\pi}\gamma_5\,(\vec{\tau}\cdot\vec{\Pi}))\,\Psi'\,, \tag{10}$$

which would yield the Lagrangian with the $\pi - N$ couplings mixed (for $\lambda\,=\,1$, one gets the pseudoscalar one).

It has recently been argued by Fearing [9] that the term (5) violates the gauge invariance of the RMC amplitude (26) derived in [1], which is somewhat misleading. Actually, its presence would violate the CVC constraint which should satisfy the hadron part of this amplitude. The validity of this constraint then guarantees the gauge invariance of the whole RMC amplitude [10,3]. On the contrary, the presence of the term (5) would violate directly the PCAC constraint for the hadron part of the RMC amplitude (26) [1].

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